II Workshop on Dynamics Numeration and Tilings (II FloripaDynSys)

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Talks

On m-minimal diffeomorphisms

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We introduce the concept of m-miinimal foliations and we apply it to partially hyperbolic diffeomorphisms. We obtain dynamical and ergodic consequences of this condition. Finally, we show that this property is abundant in several scenarios. Joint work with Felipe Nobili and Thiago Catalan.

Cohomologous potentials for the two dimensional shift

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Consider the space $X = \{0,1\}^{\mathbb{Z}}$ and $Y = \{0,1\}^{\mathbb{N}}$. It is a well known fact that given a potential $V : X \to \mathbb{R}$ there exists a potential $v : Y \to \mathbb{R}$ that is cohomologous to V, i.e., such that $v = V + h \circ \sigma - h$ where σ is the shift map (the unilateral shift in Y and the bilateral shift in X).

Now consider the space $\{0,1\}^{\mathbb{Z}^2}$ and $\{0,1\}^{\mathbb{N}^2}$; in this spaces we can define two shifts, σ_x and σ_y that corresponds to the horizontal shift and to the vertical shift, and that satisfies $\sigma_x \circ \sigma_y = \sigma_y \circ \sigma_x$. Given a potential $U : \{0,1\}^{\mathbb{Z}^2} \to \mathbb{R}$ we are able to show that there exists a potential $u : \{0,1\}^{\mathbb{N}^2} \to \mathbb{R}$ such that $u = U + g \circ \sigma_x - g + h \circ \sigma_y - h$.

This is a joint work with E. Garibaldi (UNICAMP) and E. Artuso (UFRGS), partially supported by CNPq

Geometry and Calculus of Holder Gibbs states

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We consider a transformation $T : X \to X$ which can be either the shift acting on the Bernoulli space $X = \{1, 2..., d\}^{\mathbb{N}}$ or an expanding transformation acting on the circle $X = S^1$. For a given α-Holder potential $A : X \to \mathbb{R}$ one can consider the equilibrium probability for the pressure $P(A)$.

Denote by $\mathcal{G}$ the set of equilibrium probabilities for α-Holder potentials.

In the Banach manifold $\mathcal{G}$ we introduce a natural Riemannian structure. For v in the tangent space $T_\mu \mathcal{G}$ the value $|v|^2$ is the asymptotic variance of v for $\mu = \mu_A$, where μ_A is the equilibrium probability for the normalized potential A.

We investigate the gradient flow of the entropy function $\mu \in \mathcal{G} \to h(\mu)$ and for a fixed Holder potential $B : X \to \mathbb{R}$ the gradient flow of the function $\mu \in \mathcal{G} \to G(\mu) = h(\mu) + \int B d\mu$. We also solve some interesting problems of maximization of entropy with constrains.

We show that the Riemannian structure on $\mathcal{G}$ is not compatible with the Wasserstein Riemannian structure.

The set of potentials depending of two coordinates is a subset of $\mathcal{G}$. It can be parametrized by the square $\Gamma = [0, 1] \times [0, 1] \subset \mathbb{R}^2$. The Riemannian metric restricted to the set Γ defines a manifold of positive curvature.

This is a joint work with P. Giulietti, B. Kloeckner and D. Marcon

Fractal dimensions and complexity of infinite sequences with positive entropy

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Let A be the finite alphabet $A = \{0, 1, \ldots, q-1\}$. Given an infinite word $w \in A^{\mathbb{N}}$ and $n \in \mathbb{N}$, we denote by $p_w(n)$ the number of factors of size n of w. Given a function $f : \mathbb{N} \to (0, +\infty)$, we denote by $W(f)$ the set of infinite words $w \in A^{\mathbb{N}}$ such that $p_w(n) \leq f(n), \forall n \in \mathbb{N}\}$. We associate to it the set of real numbers $C(f) = \{\sum\limits_{n\geq 0} \frac{w_n}{q^{n+1}}; \, w(x) = w_0 w_1 \cdots w_n \cdots \in W(f)\}$. We will discuss results on the combinatorics and the fractal geometry of the sets $W(f)$ and $C(f)$ for functions f of exponential growth, which correspond to infinite words with positive entropy. We will also recall previous works on the zero entropy case.

This is a joint work with Christian Mauduit.

The subsequence of Thue-Morse sequence along squares is normal

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ABSTRACT

Let $T = t(0)t(1)...t(n)... \in \{0,1\}^{\mathbb{N}}$ be the Thue-Morse sequence (i.e. the infinite word T obtained as the limit in $\{0,1\}^{\mathbb{N}}$ of the sequence of finite words $(T_r)_{r\in\mathbb{N}}$ defined by the recursion $T_0 = 0, T'_0 = 1$ and $T_{r+1} = T_r T'_r, T'_{r+1} = T'_r T_r$ for any non negative integer r).

The Thue-Morse sequence is a well known example of an almost periodic and zero entropy deterministic binary sequence.

The goal of this talk is to show that its subsequence along square numbers $(t(n^2))_{n\in\mathbb{N}}$ is normal, which mean in some sense quasi-random (joint work with Michael Drmota and Joel Rivat).

Generalized Weierstrass functions

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This work is based on the Ph.D. Thesis of Amanda de Lima. Let $f : \mathbb{S}^1 \mapsto \mathbb{S}^1$ be a $C^{2+\epsilon}$ expanding map of the circle and let $v : \mathbb{S}^1 \to \mathbb{R}$ be a $C^{1+\epsilon}$ function. Consider the twisted cohomological equation

$$v(x) = \alpha(f(x)) - Df(x)\alpha(x),$$

which has a unique bounded solution α. We show that α is either $C^{1+\epsilon}$ or a continuous but nowhere differentiable function. We show that if α is nowhere differentiable then

$$\lim_{h\to 0} \mu \left\{ x : \frac{\alpha(x+h) - \alpha(x)}{\sigma \ell h \sqrt{-\log |h|}} \leq y \right\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-\frac{t^2}{2}} dt.$$

where σ and ℓ are positive constantsand μ is the SBR measure of f. In particular α is not a Lipchitz function on any subset with positive Lebesgue measure.

Selection of ground states for double-well type potentials

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We study the zero-temperature limit of the Gibbs measures of a class of long-range potentials on a full shift of two symbols $\{0, 1\}$. These potentials were introduced by Walters as a natural space for the transfer operator. In our case, they are locally constant, Lipschitz continuous or, more generally, of summable variation. We assume there exists exactly two ground states: the fixed points 0^∞ and 1^∞. We fully characterize, in terms of the Peierls barrier between the two ground states, the zero-temperature phase diagram of such potentials, that is, the regions of convergence or divergence of the Gibbs measures as the temperature goes to zero. This is a joint work with R. Bissacot (University of Sao Paulo) and Ph. Thieullen (University of Bordeaux).

Some recent results on graded and filtered rings

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In the first part of this talk, inspired by recent developments in the theory of bimodules in crossed product von Neumann algebras, we will introduce a notion of 'strong simplicity' for group graded rings. We will give a characterization of strongly group graded rings with this property and show how this, in certain cases, can be used to describe their subrings.

In the second part of this talk, we will introduce non-associative Ore extensions which generalize the classical (associative) Ore extension construction. We will show how to encode a dynamical system, arising from a homeomorphism on a compact Hausdorff space, into a non-associative differential polynomial ring.

The second part of this talk is based on joint work with Patrik Nystedt and Johan Richter.

Purely infinite groupoid C*-algebras

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Many C*-algebras, including C*-algebras coming from tilings, have etale groupoid models. So the classification of etale groupoid C*-algebras has wide applicability. The seminal work of Kirchberg and Phillips showed that simple nuclear purely infinite C*-algebras (Kirchberg Algebras) satisfying the UCT can be classified by their ordered K-theory. It is thus interesting from a classification perspective to know which etale groupoids yield Kirchberg algebras and for this it is essential to understand precisely when an etale groupoid yields a purely infinite C*-algebra. In this talk we show that a simple etale groupoid C*-algebra is purely infinite if the nonzero positive elements of a canonical Cartan MASA are infinite. We further reduce these criteria in the case of higher rank graph groupoids. We also provide a general construction that shows we can use etale groupoids to provide concrete models for many Kirchberg algebras. We apply this construction to the groupoids associated to Bratteli diagrams and deduce that for every simple dimension group D not equal to Z, the stable Kirchberg algebra with K-theory (D, $\{0\}$) can be realised as the C * -algebra of an amenable principal groupoid. This work is joint with Lisa Clark, Adam Sierakowski and Aidan Sims

Ergodicity and elliptic dynamics in surfaces of infinite measure

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We will present new examples of cylinder flows, given by continuous skew product extensions of irrational rotations on the circle, that are ergodic and rationally ergodic along a subsequence of iterates. In particular, they exhibit law of large numbers. This is accomplished by calculating, for a subsequence of iterates, the number of visits to a region around level zero, and it is expected that such number has a gaussian distribution. This is a work in progress joint with Yuri Lima and Enrique Pujals.

Ergodic and Thermodynamic Games

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Let $T : X \to X$ and $S : Y \to Y$ be continuous maps defined on compact sets. Let

$$\varphi_i(\mu,\nu) = \int_{X\times Y} A_i(x,y)d\mu(x)d\nu(y) \ \text{ for } \ i = 1, 2,$$

where μ is T-invariant and ν is S-invariant, be *pay-off functions* for a game (in the usual sense of game theory) between players that have the set of invariant measures for T (player 1) and S (player 2) as possible *strategies*. Our goal here is to establish the notion of Nash equilibrium point for the game defined by this pay-offs and strategies. The main tools came from ergodic optimization (as we are optimizing over the set of invariant measures) or thermodynamic formalism (when we add to the integrals above the entropy of measures in order to define a second case to be explored). Both cases are ergodic versions of non-cooperative games. We show the existence of Nash equilibrium points with two independent arguments. One of the arguments works for the case with entropy, and uses only tools of thermodynamical formalism, while the other, that works in the case without entropy but can be adapted to deal with both cases, uses the Kakutani fixed point theorem. We also present examples and briefly discuss uniqueness (or lack of uniqueness). In the end we present a different example where players are allowed to collaborate. This final example show connections between cooperative games and ergodic transport.

Topological Dynamics of Generic Maps of the Cantor Space

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We will talk about some results concerning the dynamics of generic continuous maps and generic homeomorphisms of the Cantor space. We consider here the three most natural points of view, namely: the individual, the collective and the stochastic one.

Diagonal changes: a geometric generalization of continued fractions

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The (standard) *continued fraction* is an algorithm that produces for any real number x a fraction of the form $a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \dots}}$ where $a_0, a_1, \dots$ are integers. The finite truncation of this fraction are rational numbers that are exactly the so called *best approximations* of x.

It is well known that the standard continued fraction can also be seen more geometrically as a *diagonal changes* on the space of tori: it encodes the geodesic flow on $\mathrm{SL}(2,\mathbb{R})/\mathrm{SL}(2,\mathbb{Z})$ with respect to some fundamental domain.

One possible generalization to higher dimensions is via the diagonal actions on the homogeneous spaces $\mathrm{SL}(d,\mathbb{R})/\mathrm{SL}(d,\mathbb{Z})$. This approach has been proved fruitful to study diophantine properties of vectors. One difficulty to do some codings in this settings is the absence of simple fundamental domains.

We will present another generalization to surfaces of higher genus (that are so called *translation surfaces*). This generalization provides a nice coding of *Teichmüller geodesics* and gives some insight on the geometry and dynamics of translation surfaces.

KMS states on C^*-algebras associated to a local homeomorphism

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For every Hilbert bimodule over a C^*-algebra, there are natural gauge actions of the circle on the associated Toeplitz algebra and Cuntz-Pimsner algebra, and hence natural dynamics obtained by lifting these gauge actions to actions of the real line. We study the KMS states of these dynamics for a family of bimodules associated to local homeomorphisms on compact spaces. For inverse temperatures larger than a certain critical value, we find a large simplex of KMS states on the Toeplitz algebra, and we show that all KMS states on the Cuntz-Pimsner algebra have inverse temperature at most this critical value. We then reconsile our results about Cuntz-Pimsner algebra with the recent work of Thomsen.

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Posters

Fibonacci adding machine and fibered Julia sets

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The stochastic adding machine (SAM) was defined by Killeen and Taylor in [KT], as follows: let $N \in \mathbb{N} = \{0, 1, 2, \ldots\}$. By using the greedy algorithm we may write N as $N = \sum_{i=0}^{k(N)} \varepsilon_i(N) 2^i$, in a unique way where $\varepsilon_i(N) \in \{0,1\}$, for all $i \in \{0, \ldots, k(N)\}$. So, the representation of N in base 2 is given by $N = \varepsilon_{k(N)}(N) \ldots \varepsilon_0(N)$. They defined a systems of evolving equation that calculates the digits of $N+1$ in base 2, introducing an auxiliary variable "carry 1", $c_i(N)$, for each digit $\varepsilon_i(N)$, as follows:

Define $c_{-1}(N+1) = 1$ and for all $i \geq 0$, do

$$\begin{aligned} \varepsilon_i(N+1) &= (\varepsilon_i(N) + c_{i-1}(N+1)) \mod 2; \\ c_i(N+1) &= \left[\frac{\varepsilon_i(N) + c_{i-1}(N+1)}{2}\right], \end{aligned} \tag{1}$$

where $[x]$ is the integer part of $x \in \mathbb{R}^+$.

Killeen and Taylor [KT] defined a SAM considering a family of independent, identically distributed randon variables $\{e_i(n) : i \geq 0, n \in \mathbb{N}\}$, parametrized by nonnegative integers i and n, where each $e_i(n)$ takes the value 0 with probability $1-p$ the value 1 with probability p. More precisely, let N be a nonnegative integer and consider the sequences $(\varepsilon(N+1))_{i\geq 0}$ and $(c_i(N+1))_{i \geq -1}$ defined by $c_{-1}(N+1) = 1$ and for all $i \geq 0$

$$\begin{aligned} \varepsilon_i(N+1) &= (\varepsilon_i(N) + e_i(N)c_{i-1}(N+1)) \mod 2; \\ c_i(N+1) &= \left[\frac{\varepsilon_i(N) + e_i(N)c_{i-1}(N+1)}{2}\right]. \end{aligned} \tag{2}$$

Killeen and Taylor [KT] studied the spectrum of the transition operator S associated to the SAM in base 2, acting in l^∞, and they proved that the spectrum of S is equal to the filled Julia set of the quadratic map $f : \mathbb{C} \longrightarrow \mathbb{C}$ defined by $f(z) = \left(\frac{z-(1-p)}{p}\right)^2$.

In [MSV], the authors considered the SAM taking a probabilities sequence $(p_i)_{i\geq 1}$, where the probability change in each state, i.e. on the description (2) we have $e_i(N) = 1$ with probability p_{i+1} and $e_i(N) = 0$ with probability $1 - p_{i+1}$, for all $i \geq 0$, and they constructed the transition operator S related to this probabilities sequence. In particular, they proved that the spectrum of S acting in l^∞, is equal to the fibered Julia set $E := \{z \in \mathbb{C} : (\tilde{f}_j(z)) \text{ is bounded}\}$, where $\tilde{f}_j := f_j \circ \ldots \circ f_1$ and $f_j : \mathbb{C} \longrightarrow \mathbb{C}$ are maps defined by $f_j(z) = \left(\frac{z-(1-p_j)}{p_j}\right)^2$, for all $j \geq 1$.

In this work, instead of base 2, we will consider the Fibonacci base $(F_n)_{n\geq 0}$ defined by $F_n = F_{n-1} + F_{n-2}$, for all $n \geq 2$, where $F_0 = 1$ and $F_1 = 2$. Also, we will consider a probabilities sequence $(p_i)_{i\geq 1}$, instead of an unique probability p (as was done in [MS] and [UM]). Thenceforth, we will define the Fibonacci SAM and considering the transition operator S, we will prove that the Markov chain is transient if only if $\prod_{i=1}^{\infty} p_i > 0$. Otherwise, if $\sum_{i=1}^{+\infty} p_i = +\infty$, then the Markov chains is null recurrent and if $\sum_{i=2}^{+\infty} p_i F_{2(i-1)} < +\infty$, then the Markov chain is recurrent positive.

We will compute the point spectrum and prove that it is connected to the fibered Julia sets for a class of endomorphisms in $\mathbb{C}^2$. Precisely $\sigma_{pt}(S) \subset E \subset \sigma(S)$ where $E = \{z \in \mathbb{C} : (g_n \circ \ldots \circ g_0(z, z))_{n\geq 1} \text{ is bounded}\}$ and $g_n : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$ are maps defined by $g_0(x, y) = \left(\frac{x-(1-p_1)}{p_1}, \frac{y-(1-p_1)}{p_1}\right)$ and $g_n(x, y) = \left(\frac{1}{r_n}xy - \left(\frac{1}{r_n} - 1\right), x\right)$ for all $n \geq 1$, where $r_n = p_{\left[\frac{n+1}{2}\right]+1}$. Moreover, if $\liminf_{i\to+\infty} p_i > 0$ then E is compact and $\mathbb{C} \setminus E$ is connected.

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Irregular and Semi-regular Tilings of the Hyperbolic Plane

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In a very roughly way, Hyperbolic Geometry is a non-Euclidean geometry which deny the fifth Euclidean postulate, assuming that, from a point not belonging to a line, there are two lines through the point, which are parallels to the given line. One of the main property of Hyperbolic Geometry is that there exists a tiling (tessellation) of the hyperbolic plane by a regular polygon with p sides and with q other p-gons meeting in each vertex if, and only if, $(p-2)(q-2) > 4$.

Tilings of the hyperbolic plane using non-regular polygons or more than one type of regular polygons are more complexes. In this work we consider the following constructions:

i) tilings of the hyperbolic plane by copies of a semi-regular polygon with alternating angles. We study the behavior of the growth of the polygons, edges and vertices when the distance increase from a fixed initial polygon.

ii) semi-regular tilings of the hyperbolic plane, where two or more distinct regular polygons are used to tile the plane.

The Gurevic entropy for Markov shifts

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Let $\mathcal{A}$ be an alphabet, the full $\mathcal{A}$-shift is the collection of all bi-infinite sequences with symbols from $\mathcal{A}$. The full $\mathcal{A}$-shift is denoted by

$$\mathcal{A}^{\mathbb{Z}} = \{(x_i)_{i\in\mathbb{Z}} : x_i \in \mathcal{A},\ for\ all\ i \in \mathbb{Z}\}$$

The shift map on the full shift is a map $\sigma : \mathcal{A}^{\mathbb{Z}} \to \mathcal{A}^{\mathbb{Z}}$ that associated a point $x \in \mathcal{A}$ to the point whose i^{th} coordinate is $\sigma(x_i) = x_{i+1}$.

A subset $\mathcal{S} \subseteq \mathcal{A}^{\mathbb{Z}}$ is called a subshift over $\mathcal{A}$ if $\mathcal{S}$ is closed with respect to the topology of $\mathcal{A}^{\mathbb{Z}}$ and if $\mathcal{S}$ is invariant under the shift map, that is, $\sigma(\mathcal{S}) \subseteq \mathcal{S}$.

A Markov shift is a subshift which can be associated to a set $\mathcal{S}_G$ of bi-infinite walks on the edges of a countable directed graph G. The Markov shift $\mathcal{S}$ is locally compact if and only if G has finite in- and out-degree. The Markov shift is compact if and only if G is a finite graph if and only if it is a shift finite.

For a Markov shift $\mathcal{S}$, let $Per(\mathcal{S})$ be the set of periodic points of $\mathcal{S}$, that is, $Per(\mathcal{S}) = \{y \in \mathcal{S} : \sigma^n(y) = y\ for\ any\ n \in \mathbb{Z}\}$.

Let $\mathcal{S}$ and $\mathcal{T}$ be subshifts, a factor map $f : \mathcal{S} \to \mathcal{T}$ is a continuous shift commuting onto map. We say that a fiber of f on $y \in \mathcal{T}$ is the preimage set $f^{-1}(y)$. Moreover, f is said to be bounded-to-1 if there is some $M \in \mathbb{N}$ such that all fibers of f have cardinality at most M; f is finite-to-1 if all fibers are finite sets; and f is countable-to-1 if all fibers are countable sets.

The 1-point compactification $\mathcal{S}_0$ of a locally compact subshift $\mathcal{S}$ is the compact metric dynamical system which consists of the Alexandroff 1-point compactification of the shift space with the extended shift maps.

The Gurevic entropy is defined to be the topological entropy of the 1-point compactification of the subshift.

$$h_G(\mathcal{S}) = h_{top}(\mathcal{S}_0)$$

We consider the Gurevic metric the metric on $\mathcal{S}$ such that the completion of $\mathcal{S}$ with respect to this metric is $\mathcal{S}_0$.

Let $\mathcal{S}$ and $\mathcal{T}$ be transitive locally compact Markov shifts. A factor map $f : \mathcal{S} \to \mathcal{T}$ is proper if $f^{-1}(K)$ is a compact set for every compact set $K \subseteq \mathcal{T}$.

In this work, we present relationships between the Gurevic entropies of two transitive locally compact Markov shifts under some conditions on the factor maps between them.

Theorem: Let $\mathcal{S}$ be a transitive locally compact Markov shift and $\mathcal{T}$ a subshift locally compact. Let $f : \mathcal{S} \to \mathcal{T}$ a factor map. If the fiber $f^{-1}(y)$ is countable for every $y \in Per(\mathcal{T})$ then

$$h_G(\mathcal{S}) \leq h_G(\mathcal{T}).$$

In particular, if f is countable-to-1, so $h_G(\mathcal{S}) \leq h_G(\mathcal{T})$.

Theorem: Let $\mathcal{S}, \mathcal{T}$ be transitive locally compact Markov shifts and let $f : \mathcal{S} \to \mathcal{T}$ a factor map finite-to-1. Then

$$h_G(\mathcal{S}) = h_G(\mathcal{T}).$$

Theorem: Let $\mathcal{S}, \mathcal{T}$ be locally compact Markov subshifts and let $f : \mathcal{S} \to \mathcal{T}$ a factor map proper. Then

$$h_G(\mathcal{S}) \geq h_G(\mathcal{T}).$$

Theorem: Let $\mathcal{S}, \mathcal{T}$ be locally compact Markov subshifts and let $f : \mathcal{S} \to \mathcal{T}$ a factor map countable-to-1 proper. Then

$$h_G(\mathcal{S}) = h_G(\mathcal{T}).$$

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Full Semigroups of Equivalence Relations and Soficity

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Measurable dynamical systems have been extensively studied in the last decades, due to the several relations with other areas of Mathematics (in particular Operator Algebras).

In this work we consider a discrete measure-preserving equivalence relation R on a standard measure space (X,μ). The triple (X,μ,R) naturally gives rise to the *full group* $[R]$ and *semigroup* $[\![R]\!]$ of R, and a well-known result of Dye (1963) states that, in the ergodic case, $[R]$ completely determines R (up to isomorphism). In the first part of this work, we present a generalization of Dye's result to the non-ergodic case.

The second part of this work is concerned with the *sofic* property, which is a weak notion of approximability by finite structures and has been of great interest recently. It was introduced by Gromov (1999) for groups in the context of symbolic dynamics, and later given by Elek and Lippner (2010) for equivalence relations. We present a natural description of soficity in terms of full groups and semigroups.

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Non-compact group shifts over countable group alphabets

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Let $(G, \cdot)$ be a countable group. Consider the group $(G^{\mathbb{Z}}, *)$ where $*$ is the piecewise operation defined from operation $\cdot$ on G. Given $F \subset \bigcup_{n\geq 1} G^n$, define

$$X_F := \{(g_i)_{i\in\mathbb{Z}} : \ (g_i)_{m\leq i\leq n} \notin F, \ \forall m, n \in \mathbb{Z}\}.$$

We will characterize when X_F is a subgroup of $G^{\mathbb{Z}}$.

This work was supported by CNPq grants.